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National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No.NAS-5-12487

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# INITIAL RADIUS OF AN IONIZED METEOR WAKE


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19 SEPTEMBER 1966

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Geomagnetizm i Aeronomiya
Tom 6, No. 4, 713 – 716,
Izdatel'stvo "NAUKA", 1966

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## SUMMARY

Taking into account the dependence of the effective diffusion cross section of meteoric atoms in the atmosphere on meteor velocity, the initial radius of an ionized meteor wake is computed. Inasmuch as the initial widening of the wake takes place mainly at the expense of the first few path lengths of vaporized particles, the initial contribution to the initial radius of individual paths is considered. The obtained values of the initial radius $R_i$ satisfy the condition $0{,}93\, l_0 < R_н < 1{,}5\, l_0$; where $l_0$ is the length of the free path of vaporized particles for a given velocity of meteors. The computed values of the initial radius are in good agreement with the results of radar measurements of the initial radius conducted in Khar'kov, Kiyev and Jodrell Bank.

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As is shown in [1–3], the most important factor determining the observability of radiometeors is the initial radius of ionized meteor wakes.

Inasmuch as the velocity of atoms vaporizing from the surface of the meteor body is much less than the thermal velocity of atmospheric molecules $v_T$, a rapid initial widening of the meteor wake takes place prior to the establishment of equilibrium with the surrounding medium. The value of the initial radius $r_0$ of ionized meteor wakes was first appraised in [4], where a series of assumptions were however made: it was not taken into account of the effective diffusion cross section's dependence Qd on velocity of particles, and the quantity $Q_d$ was taken same as for gas-kinetic conditions; it was also assumed that prior to the establishment of thermal equilibrium, each of the vaporized particles undergoes a very large number of collisions. In subsequent works [2, 5] the computation of $r_0$ was performed taking into account the dependence of $Q_d$ on velocity. The quantity $Q_d$ was borrowed from [6]. However, the assumption of a very large number of collisions was maintained, just as in [4].

Before the velocity of a vaporized partcle is lowered to the thermal velocity, it undergoes, as an average, some 10 to 15 collisions. Taking into account the dependence of $Q_d$ on the relative velocity $\underline{v}$ of colliding particles, the successive free path lengths of meteoric atoms and ions drop fast. This is why the

* NACHAL'NYY RADIUS IONIZIROVANNOGO METEORNOGO SLEDA

initial contribution to wake's initial radius is made by the first few paths. Therefore, the assumption accepted in preceding works about the large number of collisions prior to the establishment of thermal equilibriun is not corroborated.

Assume that the velocity $v_0$ of a meteor body is directed along the axis Z (Fig. 1). The particles vaporizing from the meteor body's surface escape from it with velocities substantially lower than $v_0$ . This is why the vaporized particles form a feebly diverging beam relative to the axis Z. Inasmuch as $v_0 \gg v_T$, the atmosphere molecules may be considered as immobile. The direction of particle motion after collision is characterized by the deviation angle from the direction along which the particle moved prior to collision ($\theta$) and the azimuth $\varphi$. The mean value of $\theta$ depends on the mass ratio of colliding particles. All the values of $\varphi$ are equally probable and this is why after collision the particle will move along one of the generatrices of the cone with an aperture angle equalling $2\theta$ and the axis directed along the velocity of the particle before collision. The value of the generatrix is equal to the length $l$ of the free path, which is dependent on velocity.

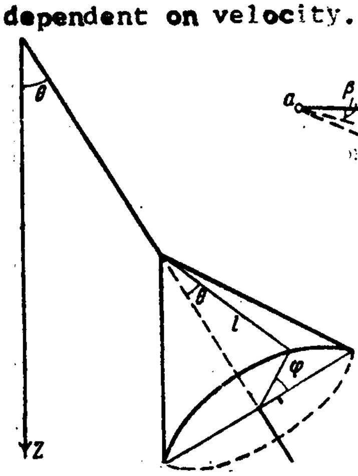


Fig. 1

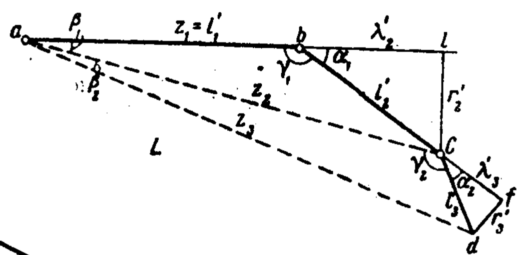


Fig. 2

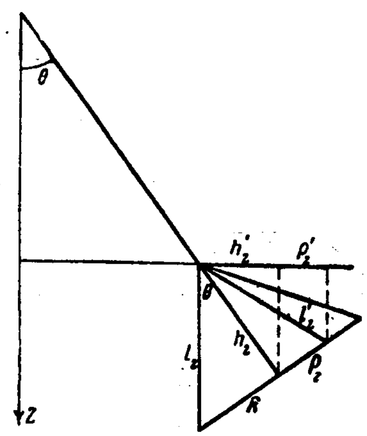


Fig. 3

After the first collision the particle will drift away in a direction perpendicular to the axis of the wake to a distance $z_1 = l_1' = l_1 \sin\theta$, after the second and the third collisions respectively to distances $z_2$ and $z_3$. For the determination of the latter we shall consider the projections $l_1', l_2', l_3'$ of cones' $l_1, l_2, l_3$ generatrices along which the particle moved after the first, second and third collisions (Fig. 2) on the plane L, perpendicular to the axis of the wake. In Fig. 2 the angles $\alpha_1$ and $\alpha_2$ are determined by azimuthal angles $\varphi_1$ and $\varphi_2$.

Fron the triangles abc and cfd we find

$$z_2^2 = z_1^2 + l_2'^2 - 2z_1 l_2' \cos\gamma_1, \qquad \gamma_1 = \pi - \alpha_1, \tag{1}$$

$$z_3^2 = z_2^2 + l_3'^2 - 2z_2 l_3' \cos\gamma, \qquad \gamma_2 = \pi + \beta_1 - \alpha_1 - \alpha_2, \tag{2}$$

$$\cos\beta_1 = \frac{l_1' + \lambda_2'}{z_2}, \quad \sin\beta_1 = \frac{r_2'}{z_2}, \quad \cos\alpha_1 = \frac{\lambda_2'}{l_2'}, \quad \sin\alpha_1 = \frac{r_2'}{l_2'},$$

$$\cos\alpha_2 = \frac{\lambda_3'}{l_3'}, \quad \sin\alpha_2 = \frac{r_3'}{l_3'}. \tag{3}$$

In order to express the quantities entering into Eqs. (1)-(3) by $l_1, l_2, l_3$, and $\theta$, let us consider the cross section of the cone by a plane passing through its axis and perpendicular to the plane L. Such a cross section for the second cone is represented in Fig. 3. Here $h_2$ is the height of the cone, R is the radius of the base; $l_2'$ is the projection of $l_2$ on the cross section plane; $h_2'$ is the projection of $h_2$ on the plane L.* From Figs 2 and 3 it is easy to obtain

$$\begin{gathered}h_2' = l_2\cos\theta\sin\theta, \qquad \rho_2' = l_2\sin\theta\cos\varphi_1\cos\theta,\\ \lambda_2' = h_2' + \rho_2' = l_2\sin\theta\cos\theta(1+\cos\varphi_1),\\ r_2' = l_2\sin\theta\sin\varphi_1, \qquad l_2'^2 = \sin_2\theta[\sin^2\varphi_1 + \cos^2\theta(1+\cos\varphi_1)^2].\end{gathered} \tag{4}$$

Analogous constructions may be performed for the third cone. Having denoted the respective values by the index 3, we obtain

$$\begin{gathered}h_3' = \frac{l_3\sin\theta\cos^2\theta(1+\cos\varphi_1)}{\sqrt{\cos^2\theta+\sin^2\theta\cos^2\varphi_1}}, \quad \rho_3' = \frac{l_3\sin\theta\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)}{\sqrt{\cos^2\theta+\sin^2\theta\cos^2\varphi_1}},\\ r_3' = l_3\sin\theta\sin\varphi_2,\\ \lambda_3' = l_3\sin\theta\left[\frac{\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)}{\sqrt{\cos^2\theta+\sin^2\theta\cos^2\varphi_1}}\right],\\ l_3' = l_3^2\sin^2\theta\left\{\sin^2\varphi_2 + \frac{[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]^2}{\cos^2\theta+\sin^2\theta\cos^2\varphi_1}\right\}\end{gathered} \tag{5}$$

With the help of (1), (3) and (4) we find

$$z_2^2 = l_1^2\sin^2\theta + l_2^2\sin^2\theta[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2] + 2l_1l_2\sin^2\theta\cos\theta(1+\cos\varphi_1). \tag{6}$$

Averaging by $\varphi_1$ we obtain

$$\bar{z}_2^2 = l_1^2\sin^2\theta + l_2^2\sin^2\theta\frac{1}{2}(1+3\cos^2\theta) + 2l_1l_2\sin^2\theta\cos\theta. \tag{7}$$

For the computation of $\bar{z}_3^2$ by formula (2) it remains to determine and average by azimuths the second and third terms of this formula. With the aid of (3) and (5) it is easy to show that

$$-2z_2l_3'\cos\gamma_2 = \frac{2}{l_2'}\{\lambda_3'[\lambda_2'(l_1'+\lambda_2')+r_2'^2] - r_3'[r_2'(l_1'+\lambda_2')-\lambda_2'r_2']\}. \tag{8}$$

Inasmuch as $\bar{r}_3' = l_3\sin\theta\overline{\sin\varphi_2} = 0$, averaging (8) by $\varphi$ we obtain

$$\overline{-2z_2l_3'\cos\gamma_2} = \frac{2}{l_2'}(\overline{\lambda_3'\lambda_2'l_1'} + \overline{\lambda_3'\lambda_2'^2} + \overline{\lambda_3'r_3'^2}). \tag{9}$$

We shall compute each of the terms in the right-hand of (9) separately

$$\begin{gathered}\frac{\overline{\lambda_3'\lambda_2'l_1'}}{l_2'} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2\times\\ \times\frac{l_1l_3\sin^2\theta\cos(1+\cos\varphi_1)[\cos^2\theta(1+\cos\varphi_1)+\cos\varphi_2(\cos^2\theta-\sin^2\theta\cos\varphi_1)]}{\sqrt{(\cos^2\theta+\sin^2\theta\cos^2\varphi_1)[\sin^2\varphi_1+\cos^2\theta(1+\cos\varphi_1)^2]}} =\\ = \frac{l_1l_3\sin^2\theta\cos^3\theta}{2\pi}(I_1+2I_2+I_3),\end{gathered} \tag{10}$$

* the same goes for $\rho_2$.

where
$$I_k = \int_{-\pi}^{\pi} \frac{\cos^{k-1}\varphi_1 d\varphi_1}{\sqrt{a_1\cos^4\varphi_1 + a_2\cos^3\varphi_1 + a_3\cos^2\varphi_1 + a_4\cos\varphi_1 + a_5}} \tag{11}$$

Here
$$a_1 = -\sin^4\theta, \quad a_2 = 2\cos^2\theta\sin^2\theta, \quad a_3 = \sin^2\theta, \quad a_4 = 2\cos^4\theta,$$
$$a_5 = \cos^2\theta(1+\cos^2\theta),$$

$$\frac{\overline{\lambda_3'\lambda_2'^2}}{l_2'} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \times$$
$$\times \frac{l_2 l_3 \sin^2\theta[\cos^2\theta(1+\cos\varphi_1) + \cos\varphi_2(\cos^2\theta - \sin^2\theta\cos\varphi_1)]\cos^2\theta)1+\cos^2\varphi_1)}{\sqrt{(\cos^2\theta + \sin^2\theta\cos^2\varphi_1)[\sin^2\varphi_1 + \cos^2\theta(1+\cos\varphi_1)^2]}} =$$
$$= \frac{l_2 l_3 \sin^2\theta\cos^4\theta}{2\pi}(I_1 + 3I_2 + 3I_3 + I_4). \tag{12}$$

$$\frac{\overline{\lambda_3' r_3'^2}}{l_2'} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \times$$
$$\times \frac{l_2 l_3 \sin^2\theta\sin^2\varphi_1[\cos^2\theta(1+\cos\varphi_1) + \cos\varphi_2(\cos^2\theta - \sin^2\theta\cos\varphi_1)]}{\sqrt{(\cos^2\theta + \sin^2\theta\cos^2\varphi_1)[\sin^2\varphi_1 + \cos^2\theta(1+\cos\varphi_1)^2]}} =$$
$$= \frac{1}{2\pi} l_2 l_3 \sin^2\theta\cos^2\theta(I_1 + I_2 - I_3 - I_4). \tag{13}$$

Substituting (10), (12) and (13) into (9), we shall obtain

$$-2z_2 l_3'\cos\gamma_2 = \frac{1}{\pi} l_3 \sin^2\theta\cos^2\theta[(l_1\cos\theta + l_2\cos^2\theta + l_2)I_1 +$$
$$+ (2l_1\cos\theta + 3l_2\cos^2\theta + l_2)I_2 + (l_1\cos\theta + 3l_2\cos^2\theta - l_2)I_3 + (l_2\cos^2\theta - l_2)I_4] \tag{14}$$

The mean value of the third term in formula (2) is

$$\overline{l_3'^2} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 l_3^2 \sin^2\theta \Big\{ \sin^2\varphi_2 +$$
$$+ \frac{[\cos^2\theta(1+\cos\varphi_1) + \cos\varphi_2(\cos^2\theta - \sin^2\theta\cos\varphi_1)]^2}{\cos^2\theta + \sin^2\theta\cos^2\varphi_1}\Big\} =$$
$$= l_3^2\sin^2\theta\left[\frac{1}{2} + \frac{3}{2}\cos^2\theta + \left(\operatorname{ctg}^2\theta - \cos^2\theta + \frac{\sin^2\theta}{2}\right)(1-\cos\theta)\right]. \tag{15}$$

Taking into account (7), (14) and (15) we shall finally obtain

$$\bar{z}_3^2 = \sin^2\theta\Big\{l_1^2 + \frac{1}{2}l_2^2(1+3\cos^2\theta) + 2l_1 l_2\cos\theta + \frac{l_3\cos^2\theta}{\pi}\times$$
$$\times[(l_1\cos\theta + l_2\cos^2\theta + l_2)I_1 + (2l_1\cos\theta + 3l_2\cos^2\theta + l_2)I_2 +$$
$$+ (l_1\cos\theta + 3l_2\cos^2\theta - l_2)I_3 + (l_2\cos^2\theta - l_2)I_4] +$$
$$+ l_3^2\left[\frac{1}{2} + \frac{3}{2}\cos^2\theta + \left(\operatorname{ctg}^2\theta - \cos^2\theta + \frac{1}{2}\sin^2\theta\right)(1-\cos\theta)\right]\Big\}. \tag{16}$$

By substituting $\cos\varphi_1 = x$, the integrals $I_k$ entering into formula (16) will be brought to integrals of the form

$$\int_{-1}^{1} f_k(x)\,dx/\sqrt{1-x^2}. \tag{17}$$

Inasmuch as the integrand in (17) has a peculiarity for $x = \pm 1$, we utilized for the numerical integration the Gauss-Mahler formula, not including the values of the function at interval ends.

In case of equality of colliding particle masses the mean value is $\overline{\cos\theta} = 2/3$ (see [4]). The velocity of the vaporized particle after $\varkappa$ collisions is, as an average [4]

$$v_\varkappa = v_0 \exp(-\alpha\varkappa), \tag{18}$$

and for equal masses of particles $\alpha = 0.4$.

At meteor velocities for various pairs of colliding particles the quantity $Q_d$ is computed in the works [6–8]. The obtained dependence of $Q_d$ on velocity may be approximated by formula

$$Q_d = C / v, \tag{19}$$

where C is a constant. For collisions of typical meteor atoms with atmosphere molecules [8] $C \approx 1{,}7\cdot 10^{-9}$ $см^3$/sec. As pointed out in [6–8], the effective diffusion cross section of ions for meteor velocities differs little from that for neutral particles. Then the free path length dependence on velocity will have the form

$$l = v / nC. \tag{20}$$

Here n is the number of atmosphere molecules per volume unit. From (16) – (18) and (20) we find $\bar{z}_3 = 0{,}93\, l_0$, where $l_0$ is the length of the free path to the first collision. Obviously, the value of wake's radius for $\varkappa > 3$ will be somewhat greater than $\bar{z}_3$. Thus the quantity $z_3$ may be taken as the minimum estimate of wake's initial radius $r_{min}$. To obtain the maximum estimate $r_{max}$ we shall add to $\bar{z}_3$ the sum of all the remaining paths prior to settlement of thermal equilibrium with the atmosphere. With the help of formulas (18) and (20) we shall obtain $r_{max} = 1{,}5\, l_0$. Therefore, the real value of the initial radius is comprised within the limits

$$0{,}93\, l_0 < r_0 < 1{,}5\, l_0. \tag{21}$$

From radar measurements of the initial radius of ionized meteor wakes by the method of parallel meteor observations in two wavelengths it was found [2], that for meteor velocity $v_0 = 32$ km/sec at 95 km altitude, the mean value is $r_0 = 0.8$m. From (20) and for $v_0 = 32$ km/sec we obtain $l_0 = 0.64$m (the density of the atmosphere was borrowed from [10]). Then (21) takes the form: $0{,}60 < r_0 < 0{,}96$ м. According to data of [9] for $v_0 = 41$ km/sec at 96 km, the mean value is $r_0 = 1.0$ m. According to (21) we find in this case $0{,}97 < r_0 < 1{,}57$ м. Finally, according to [1] a feeble dependence of $r_0$ on the height $(r_0 \sim l_0^{0,3})$ has been obtained alongside with the total absence of dependence on velocity. As is shown in [11], such results are explained by imperfection of measurement methods. According to that work, among the values obtained in [1], the most confidence is deserved by the value $r_0 = 1.55$ m, related to the altitude of 101 km. Admitting that the mean value is $v_0 = 40$ km/sec, we shall obtain $1{,}6 < r_0 < 2{,}6$ м.

**** THE END ****

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Institute of Applied Geophysics — Received on 26 July, 1965.


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Contract No. NAS-5-12487
VOLT TECHNICAL CORPORATION
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Translated by ANDRE L. BRICHANT

on 15 September 1966


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